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NUMERICAL THERMAL ANALYSIS FOR AN IDEAL CRYOGENIC REGENERATOR

Natheer Almtireen[1], Jürgen J. Brandner[1], Jan G. Korvink*[1]
[1]Karlsruhe Institute of Technology (KIT), Institute of Microstructure Technology (IMT) Eggenstein-Leopoldshafen, 76344, Germany.
Natheer.Almtireen@kit.edu, Juergen.Brandner@kit.edu, Jan.Korvink@kit.edu

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ABSTRACT

Regenerative cryocoolers like Stirling, Gifford-McMahon and pulse tube cryocoolers possess great merits like small size, low cost, high reliability and good cooling capacity. These merits led them to meet many IR and superconducting based applications requirements. The regenerator is a vital element in these closed-cycle regenerative cryocoolers, whereas their performance depends strongly on the effectiveness of the regenerator. This paper presents a one-dimensional numerical analysis for the idealised thermal equations of the matrix and the working gas inside the regenerator. The algorithm predicts the temperature profiles for the gas during the heating and cooling periods along with the matrix nodal temperatures. It examines the effect of the regenerator length and the volumetric flow rate on the performance of the regenerator.

1. INTRODUCTION

The growth in many low-temperature applications such as IR imaging, superconductivity-based systems and many others has led to huge development in cryocoolers over the last few decades. One interesting application is micro-cryocoolers for IR imaging systems [1]. In general, any good design of such micro-cryocooler requires thorough study of each component working principle, in which the cryocooler performance is highly dependent on the efficiency of each component. The regenerator is a vital component in any closed-cycle regenerative cryocooler such as Stirling, Gifford-McMahon (G-M) and pulse tube machines. The regenerator acts as thermal storage component that transmits the pressure signal from the compressor to the pulse tube and the other cryocooler components. It is mainly constructed of hollow thin tubes filled with mesh screens and/or metallic spheres etc. The working principle involves thermal energy exchange between the working gas and the matrix material. To maintain considerable refrigeration, this matrix takes away the heat from the incoming gas during the heating phase, and delivers it back for the returning gas during the cooling phase.

In 1816, Stirling introduced the regenerative heat exchanger in hot air engine [2]. In 1927, Nusselt presented a mathematical analysis for a special case regenerator with infinite matrix heat capacity [2]. Iliffe used the ideal analyses of several German authors to perform a first order graphical numerical analysis that accounts for time variations of the matrix temperature in 1948 [3]. One of the major advances happened in 1960; when Gifford and McMahon developed a small regenerative cryogenic refrigerator to cool infrared detectors and maser amplifiers, which moved cryogenics out of the laboratory to the industry [2]. Kuriyama et al. [4] used rare-earth regenerative materials to achieve ultra-low temperatures with a G-M multi-stage cryocooler.

* Corresponding author

Numerical analyses and experimental work are very significant to give better interpretation for the thermal interaction in the regenerator between the working gas and the matrix material. Radebaugh [5] introduced an optimization procedure for designing regenerators. The REGEN [6] series software developed at the National Institute of Standards and Technologies (NIST) was used by many researchers to study the regenerator performance for different matrix materials. Other available software packages include SAGE® and DELTAE which solve the conservation equations and, hence, describe the regenerator performance are also often used. These numerical analyses have different levels of complexity depending on the assumptions employed. Complex numerical models account for pressure drop, void volume, conduction losses, pressure variations, time-varying mass flow rates, time-varying gas inlet temperature and temperature-dependent thermal properties. This paper numerically solves the idealized thermal characteristic equations for the working gas and the matrix based on finite difference techniques to predict the output temperature profiles for the working gas and the matrix material.

2. IDEAL REGENERATOR SYSTEM

2.1 Model and Numerical Solution

The mathematical model used for this publication is based on simplified thermal energy conservation for the working gas and the matrix as suggested by Ackermann [2]. The one-dimensional model is built on various assumptions: first, the mass flow and the working gas pressure through the regenerator are constants and their magnitudes are equal during heating and cooling periods with no gas flow-mixing during these periods. Second, the boundary temperature conditions for the working gas are constants for heating and cooling periods. Third, the fluid stored thermal energy is zero with zero longitudinal matrix thermal conductivity. As a result, the matrix and fluid thermal equations are, respectively:

$$h\delta A_s(T_g - T_m) = (\rho c_p \delta V)_m \frac{\partial T_m}{\partial t} \tag{1}$$

$$h\delta A_s(T_g - T_m) = -(\rho c_p \delta V)_g u_x \frac{\partial T_g}{\partial x} \tag{2}$$

Where h, A_s, T_m, T_g , ρ, u_x, V *and* c_p are the convection heat transfer coefficient, matrix heat transfer area, matrix temperature, gas temperature, the density of the gas, the velocity of the gas, the occupied volume and constant pressure specific heat capacity, respectively. The left-hand sides for both equations represent the convection heat transfer between the matrix material and the working gas, while the right-hand sides in equations (1) and (2) represent the varying built-up heat in the matrix and the working gas, respectively. Equations (1) and (2) are then expressed in finite-element difference form as proposed by Ackermann [2]; here the regenerator is divided into spatial elements and the heating and cooling periods are divided into small time steps. Fig. 1 shows the proposed numerical schematic for the regenerator during heating and cooling periods.

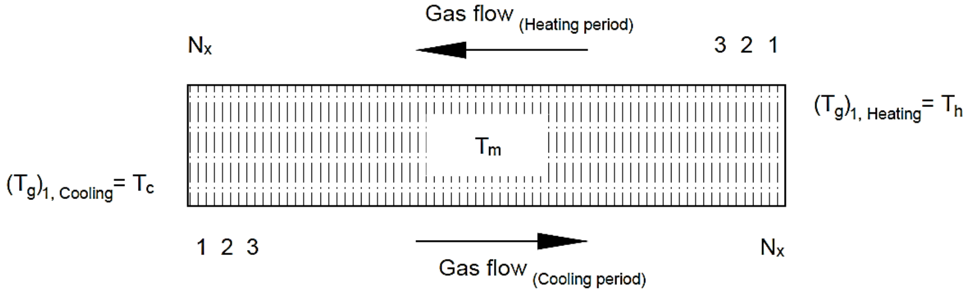

Figure.1 Numerical schematic for the regenerator during heating and cooling periods, where Nx and Nt are the total number of spatial and time elements, Th and Tc are the hot and cold temperature levels.

The model is then converted to two linear algebraic equations, which are used to compute the nodal temperatures for the matrix and the gas and, hence predict the regenerator performance. These equations can be summarised as:

$$(T_g)_{i+1}^{j} = (T_g)_i^j - K_1((T_g)_i^j + (T_m)_i^j) \tag{3}$$

$$(T_m)_i^{j+1} = (T_g)_i^j + K_2((T_g)_i^j + (T_m)_i^j) \tag{4}$$

i and *j* are the number of spatial and time nodes, K_1 and K_2 are constants (see [2]). These equations are solved in iterative fashion for each spatial and time nodes during both the heating and cooling periods. The numerical scheme assumes linear distribution of temperature between T_h and T_c across the matrix. Also, the fluid is assumed to enter the regenerator at T_h for the heating period and T_c for the cooling period. The produced MATLAB® code calculates the nodal temperatures for the matrix and gas during the heating period. The output spatial nodal temperatures for the matrix at the end of the heating period are then used as the input spatial nodal temperatures for the matrix at the start of the cooling period such as:

$$(T_m)_{1:N_x}^{1}/_{Cooling} = (T_m)_{N_x:1}^{N_t}/_{Heating}$$

For the algorithm to converge into a solution, an increased number of time steps is needed, and the number of spatial nodes should be larger than the total number of heat transfer units (NTU). More spatial and time steps are needed to achieve an accurate solution. The relation between the regenerator inefficiency (Ie) and both the spatial and time steps is discussed in section (3).

2.2 Model Convergence

The model converge to solution under two conditions; During the heating cycle, the final matrix temperature is not higher than the outlet fluid temperature. During the cooling period; the final fluid temperature is not higher than the outlet fluid temperature, this can be summrized as:

$$Condition1: \qquad (T_m)_{N_x}^{N_t}/_{Heating} \ngtr (T_f)_{N_x}^{N_t}/_{Heating}$$

$$Condition2: \qquad (T_f)_{N_x}^{N_t}/_{Cooling} \ngtr (T_m)_{N_x}^{N_t}/_{Cooling}$$

Where the maximum time interval, for each successive nodal calculation, is achieved when the the output matrix and fluid temperatuires are equal. If the outlet temperatures go beyond that, the crossing of the outlet temperatures will cause sign reversing the inlet temperature difference for the next calculation cycle causing a divergence for the numerical solution. These two conditions are applied to equations (1) and (2) to deduce the convergence criteria.

2.3 Model Application

The model is applied to a generator of 7.5 mm and 30 mm length and diameter, respectively. The regenerator matrix is made of phosphor bronze #150 woven wire mesh; the screen wire-diameter is 63μm and the screen has the same value as the regenerator diameter, if perfect stacking is assumed; the number of screens is set to 238. The cold and hot ends temperatures are set to 80K and 300K, respectively. The relation between the volumetric flow rate and the (Ie) is studied, also the effect of reduced regenerator length on (Ie) is examined.

3. RESULTS & DISCUSSION

Fig. 2 shows the gas and matrix temperature profiles during heating and cooling periods. It can be noticed that the temperature of the matrix material is increasing for all spatial elements during the heating period while the hot gas temperature drops gradually as it crosses the regenerator heating up the matrix. Accordingly, the cold gas is heated up as it passes the matrix resulting in cooling the matrix. Fig. 2 (right) shows that the algorithm converges to a solution where the difference in temperature between the matrix and gas is minimal; this happens after increasing the number of time and space nodes.

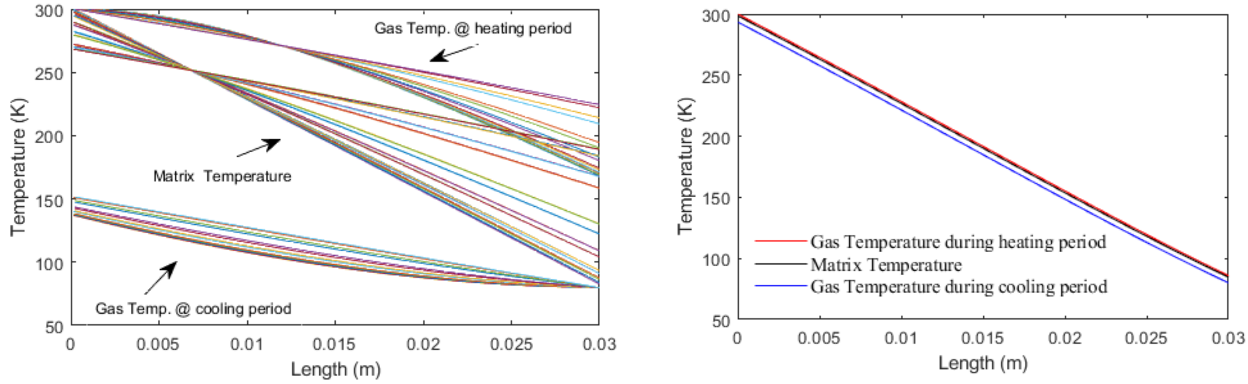

Figure.2 Left: matrix and gas temperature variations during heating and cooling period, right: the final temperature profiles as they converge to solution with increasing Nx and Nt.

The dimensional and parametric analysis are of huge importance for any efficient regenerator design. The relations between the volumetric flow rate and regenerator length, on one hand, to the regenerator inefficiency, on other hand, are illustrated in Fig. 3. These are based on the ideal case interpretation of the thermal interaction between the working gas and the matrix material, hence, it does not consider conduction or viscous losses. Moreover it assumes that the thermal properties for both the matrix and the working gas are constant with temperature variations inside the tube.

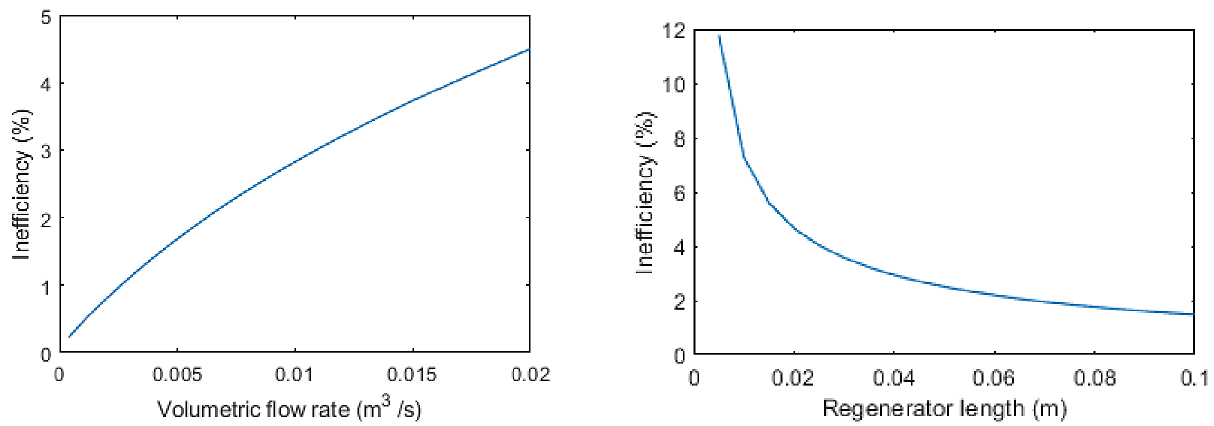

Figure.3 Left: effect of increasing volumetric flow rate on Ie, right: the relation between the regenerator length and Ie.

For 0.001 m^3/s and NTU_{Total} (total number of heat transfer units) of 63, *Nx = 5000* and *Nt =200* the (Ie) was found to 0.21% and $39.8 \leq R_e \leq 365$ where R_e is the Reynolds number. A regenerator thermal loss of ~ 0.39 Watts was obtained.

4. CONCLUSIONS

A MATLAB® code has been developed to study the thermal interaction between the working gas and the matrix material in a regenerator element, which is conventionally used for a closed-cycle regenerative cryocooler. The presented algorithm is based on the ideal regenerator thermal equations. This study illustrated the unsteady behaviour of the gas and matrix inside the regenerator, it is observed; that increasing the number of spatial and time nodes enhances the convergence of the algorithm. The paper investigates the effect of regenerator length and volumetric flow rate on the performance of the regenerator. It is found that these parameters have significant effect on the inefficiency (*Ie*) of the regenerator. Future work will expand the model to consider other substantial losses.

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