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EXPERIMENTAL EVIDENCE OF SUBSONIC CHOKING IN MICROCHANNEL SLIP FLOW

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KEY WORDS

Choked gas, High speed, Micro-scale, Compressible, Poiseuille number, Rarefied gas flow

ABSTRACT

Choking in microchannels for rarefied gas flows has been rarely observed experimentally in the existing literature. The procedure of fabricating the three-dimensional straight microchannel with rectangular cross-section of aspect ratio (width/height = 0.49) using soft lithography technique has been explained in detail. The mass driven flow of nitrogen through the channel is performed with the aim of determining friction factor across the channel. The measurements of static pressure, temperature and volume flow rate have performed at the inlet and outlet of the channel. The dependence of Poiseuille number (product of friction factor and Reynolds number) on Mach number for choking conditions has not been reported as per authors' best knowledge. This work claims an inverse proportionality of Poiseuille number with exit Mach number ($0.43 < Ma_o < 0.99$) in slip flow regime ($4.04 \times 10^{-3} < Kn_o < 7.04 \times 10^{-3}$). The results have been validated with the analytical solution from standard compressible flow theory. The flow appears to be choked before exit Mach number of unity, due to which it is referred to as subsonically choked phenomenon. This research could be applicable in high speed gas leakages through cracks formed in space shuttles and satellites manoeuvring in upper atmospheric rarefied regions.

1. INTRODUCTION

Transportation of natural gas over long distances is carried out by commercial pipelines, where compressible isothermal flow with friction has great significance. Other engineering applications where compressible gases flow in constant area ducts are transport of gases in chemical process and stationary power plants, high vacuum technology, aircraft propulsion engines and flow machineries [1]. The flow of high speed gas in upper atmosphere in case of cracks occurring in moving space-shuttles and satellites is another promising application area of the current research.

Effect of compressibility is important in rarefied choked flow inside microchannels, where the exit conditions play a significant role on flow physics. A threshold value for the back pressure exists in the continuum flow regime, below which the flow is choked. However, the back pressure always affects the flow in the rarefied regime. As the degree of rarefaction increases, the effect of back pressure is realized more in the upstream direction. When the value of back pressure is zero, the flow rate achieved is the maximum [2]. At a fixed pressure of upstream tank, if the pressure of downstream tank is decreased, value of mass flow rate across the channel increases first and then slowly achieves a saturation value. The process of saturation of the mass flow rate is called choking. On reducing the pressure of upstream tank, the difficulty in attaining choked state increases.

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The few rarefied experimental studies by Yao. et al. [3] and Harley et al. [4] include inlet pressure values more than 1 atmosphere and outlet pressure equal to 1 atmosphere. Our work includes low pressure flows of inlet pressure (P_i) values less than 1 atmosphere, with pressure ratio (P_i/P_o) ranging from 8.72 to 8.17. The maximum mass flow rate is obtained when inlet pressure shoots 1 atmosphere. Hence, the range of parameters is entirely different from the existing experimental data. The chosen aspect ratio of 0.49 is similar to the three-dimensional numerical study by Garg and Agrawal [5].

The current work allows measurements of static pressure, temperature and mass flow rates at the inlet and outlet of 3D microchannel. It investigates the influence of Reynolds number ($91.21 < Re < 364.84$) on friction factor as well as the influence of outlet Mach number ($0.43 < Ma_o < 0.99$) on Poiseuille number (fRe). The Knudsen number at outlet (Kn_o) varies from 4.04×10^{-3} to 7.04×10^{-3}, hence the flow lies in early slip regime ($0.001 < Kn < 0.1$).

2. EXPERIMENTAL SETUP

This section explicates the fabrication procedure of the three-dimensional microchannel. The experimental set-up of mass driven flow has been illustrated with all the details of instrumentation.

2.1 Fabrication of Mircochannel

We choose Poly(dimethylsiloxane) (PDMS) manufacturing technique here due to its ease of fabrication and low cost. A three-dimensional (3D) microchannel is fabricated using soft-lithography with PDMS and bonding with glass. A double side polished, 2 inch silicon wafer of 100 orientation and 4-7 ohm resistivity undergoes RCA 1 and 2 cleaning for removal of organic and metallic contaminants, respectively. An oxide layer of 0.7 to 1 µm thickness is deposited on Si wafer by keeping it in wet oxidation furnace for two hours. The thickness of oxide layer is measured using ellipsometer. In order to achieve feature depth of about 100 µm on the oxidised wafer, a solution of negative photoresist (SU-8 2100) is spun for 30 s at 3000 rpm. This coated wafer is pre-baked for 5 minutes at 338 K and 30 minutes at 368 K. A mask with the required microchannel feature design is aligned on the coated wafer to expose SU-8 to UV light for 90 s using double sided aligner. After exposure, SU-8 developer is utilized to make a mold and eliminate unexposed SU-8. The wafer is washed using iso-propyl alcohol and post baked at 338 K for 5 minutes and 368 K for 30 minutes.

This photolithography process leads to the creation of SU-8 mold whose negative replica has to be made in PDMS. The latter in mixed with a curing agent in 20:1 ratio. This mixture is deaerated in a vacuum dessicator. This mixture is poured over the mold and allowed to settle in an oven heated at 338 K for 15 minutes. A casting of required microchannel is created inside PDMS which is peeled off from SU-8 mold. 2 mm holes are punched through the inlet and outlet reservoirs of this fabricated casting. The casting is bonded with quartz glass using a mixture of PDMS and curing agent (ratio 10:1). A strong bond is formed between the two PDMS substrates by heating them for about 2 hours at 368 K. T-shaped silicon (Si) connectors are placed inside punched holes and Si tubings are sealed on both ends, which complete the process of making pressure/temperature ports. One end of T-shaped connector goes to pressure gauge/thermocouple and other end towards the gaseous flow loop. The height (H) of microchannel is 201.34, µm, width (W) 98.47 µm and length (L) 30,000 µm. Therefore, the aspect ratio ($\alpha = W/H$) of this 3D microchannel is 0.49. The channel roughness value is 0.44 µm.

2.2 Method of Measurement

The schematic and actual image experimental set-up is shown in Fig. 1. A 47 liters nitrogen cylinder (7 cubic meters gas capacity with 99.99% purity of gas) is connected to a dual stage pressure reduction regulator and filter. The filter is followed by a calibrated mass flow controller (MFC I) MKS, type 1179A of 200 sccm which controls flow over a range of 2-100% of full scale flow. The gas then settles down in the upstream tank connected to a calibrated MKS 626C Baratron absolute capacitance manometers of 1000 Torr range before entering the microchannel through Si tubings. These long tubings are connected before and after the microchannel to prevent any sudden contraction effect at inlet of the microchannel and sudden expansion effect at the outlet. Inlet and outlet pressures are measured using calibrated pressure gauges of 1000 Torr and 500 Torr respectively. The accuracy is 0.25% of reading.

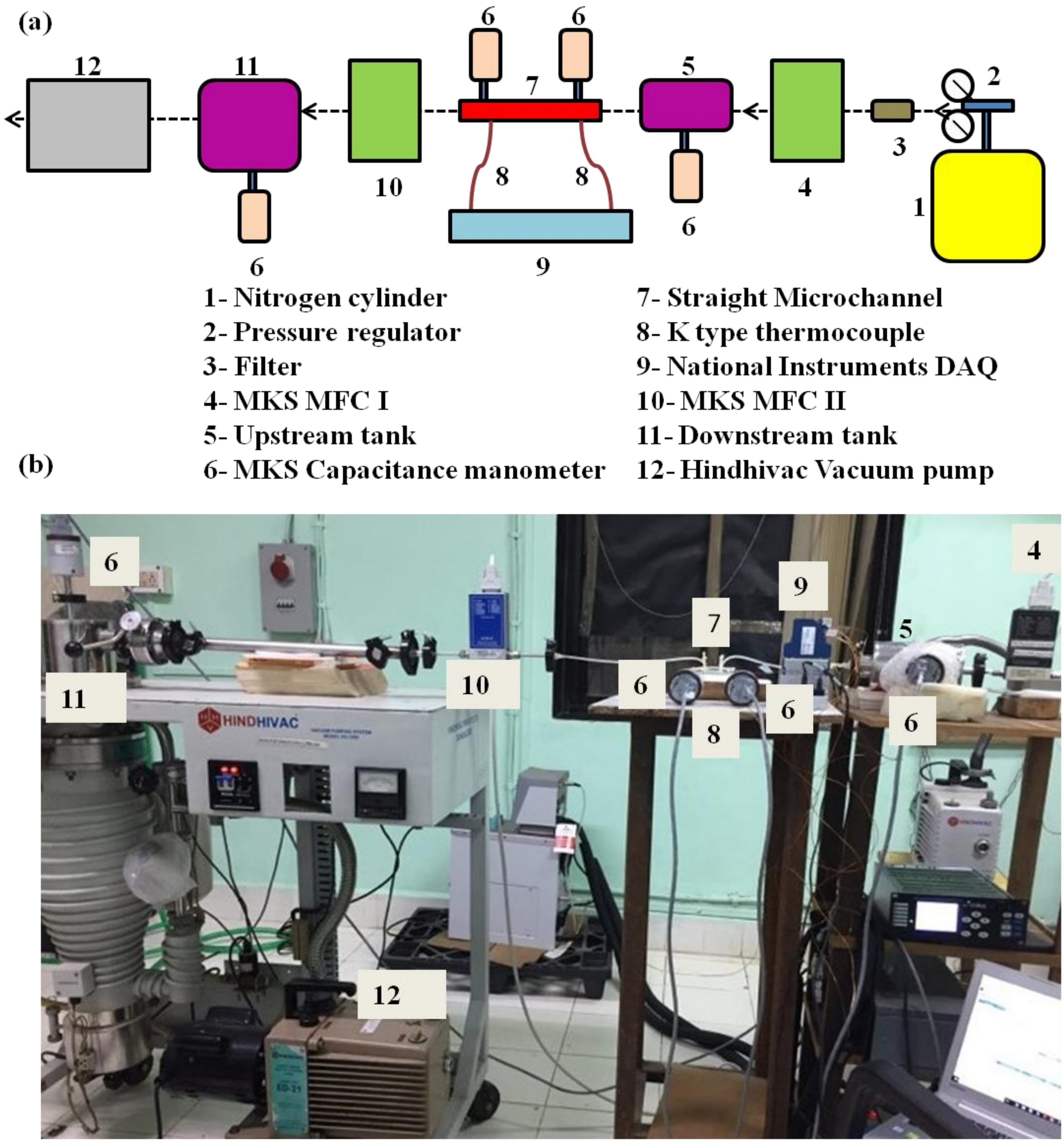

Figure 1 (a): Schematic of experimental set-up, **(b)**: Actual image of set-up with real-time measurements

Another calibrated mass flow controller (MFC II) MKS, type GE50A of 1000 sccm flow rate is attached downstream of channel to meter the flow coming out of the microchannel. The mass flow rate and absolute pressure readings are recorded through MKS 946 vacuum system controller. The flow finally enters the downstream tank, where pressure is monitored through 20 Torr gauge. This is a mass driven system where gas is driven using Hindhivac rotary vacuum pump (ED-21) which has pumping capacity of 10^{-1} Pa. The leakage in the system is 0.14 sccm which is 1% of minimum flow rate passed through the channel. MFC I controls the flow before entering the microchannel and MFC II meters the flow after exiting the microchannel. MFC II reading reflects the actual flow rate that is being pushed by the microchannel in choked conditions of the system; hence it is more reliable for the purpose of calculations.

3. RESULTS AND DISCUSSIONS

Mass flow rate and absolute pressures at inlet and outlet of the microchannel are measured for scrutinising frictional resistance. It is observed that pressure drop increases with mass flow rate. The Reynolds number, Knudsen number and Mach number are calculated by Eq.(1), Eq. (2) and Eq.(3) respectively.

3.1 Influence of Reynolds number on friction factor

Friction factor (f) is derived using momentum theorem (Eq.(3)) [6], accounting for the flow acceleration effect in compressible flows. The friction factor values are compared against one-dimensional analytical solution by Shapiro [1] and White [7] (Eq.(4)) for isothermal compressible flows [1, 7]. There is an excellent agreement which gives us confidence in our measurements (Fig. 2). This validates the presented experimental results. Friction factor decreases monotonically with Reynolds number. Uncertainty analysis of friction factor (δf) shows a maximum error value of 15.8%, resulting mainly from uncertainty in mass flow rate [8]. The size of error bars reduces as the mass flow rate increases (Fig. 2).

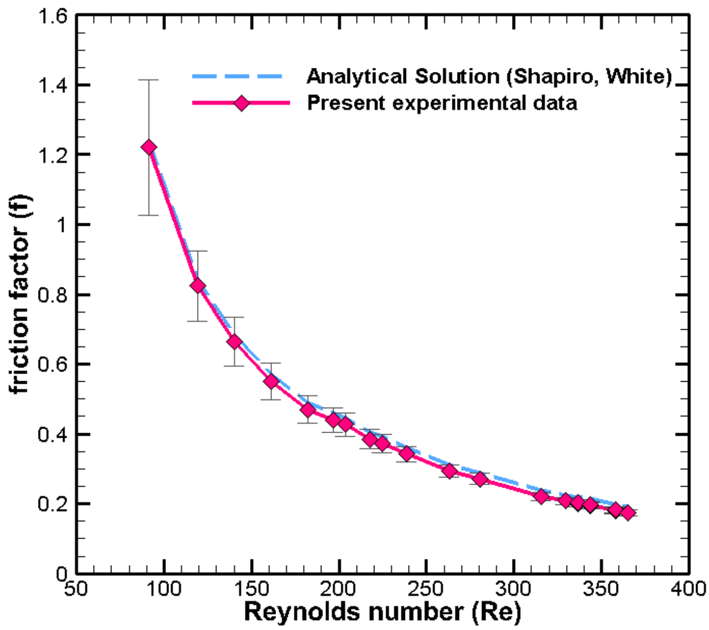

Figure 2: Validation of friction factor results with analytical solution by Shapiro [1] and White [7]

To check for the isothermal flow behaviour, K-type thermocouples are utilized for measuring temperature at inlet and outlet of the microchannel. Both the locations indicate a temperature of 298 K with a maximum deviation of 0.72 K. Therefore, an experimental validation of isothermal flow behaviour is also achieved.

3.2 Influence of Mach number on Poiseuille number

Outlet Mach number (Ma_o) (Eq.(3)) is always greater than 0.4 and reaches a maximum of 0.99 (Fig. 3) at flow rate of 52 sccm. Therefore, flow is highly compressible ($Ma_o > 0.3$) throughout the Reynolds (Eq.(1)) number ($91.2 < Re < 361.8$) range. When MFC II displays a change of flow rate from 51 to 52 sccm, there is a change in inlet pressure reading from below 1 atmosphere to above 1 atmosphere. Due to which, MFC II is not able to allow flow rate more than 52 sccm, whatever be the flow rate of MFC I. Therefore, 52 sccm is the maximum flow rate that could be allowed through this microchannel, at which the outlet Mach number value is 0.99.

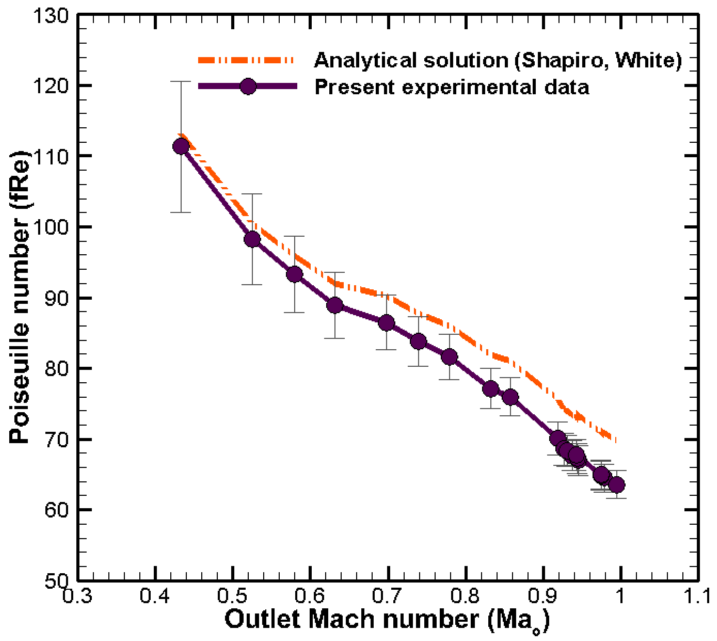

Figure 3: Comparison of Poiseuille number versus Outlet Mach number results with analytical solution by Shapiro [1] and White [7]

Fig. 3 displays the influence of Ma_o on Poiseuille number (fRe), the product of friction factor and Reynolds number. fRe is inversely proportional to Ma_o. The magnitude of fRe becomes almost half from 111.35 to 63.55 as outlet Mach number varies from 0.4 to 0.99. The results are in good agreement with analytical solutions of Shapiro [1] and White [7] at the lower Mach numbers and deviate from analytical solution towards higher Mach numbers. This means the flow physics of 3D microchannel cannot be predicted well near choked state using one-dimensional analytical solution [1, 7]. The uncertainty in Poiseuille number (δfRe) decreases from 8.25% to 2.99% as the Ma_o increases. Hence, the measured data is more reliable in the choked conditions.

According to Shapiro [1], one-dimensional (1D) adiabatic flow chokes at the outlet Mach number (Ma_o) of value unity, whereas 1D isothermal flow in a long duct chokes at $Ma_o = 1/\sqrt{\gamma} = 0.845$. In our measurements, $Ma_o = 0.832$ is found to occur at the flow rate of 37.5 sccm (1 sccm = 1.872×10^{-8} kg/s) and $Ma_o = 0.857$ occurs at the flow rate of 40 sccm. Therefore, the flow might be choked in between 37.5 sccm and 40 sccm. We check for any occurrence of subsonic choking in the flow. Initially MFC I displays a lower value than MFC II indicating leakage into the system at lower pressures. At the flow rate of 26 sccm, MFC I however displays a greater value than MFC II. This indicates that microchannel is not able to allow the entire gas sent to it through MFC I. Based on this, we deduce that the flow is choked under this condition. Since, Ma_o at this mass flow rate is 0.679 which is lower than theoretical limiting Mach number of 0.845, the flow is subsonically choked at 26 sccm. This might occur because of the three-dimensional nature of our microchannel. In a 1D flow, the velocity profile varies only in x-direction, flow behaves more like a slug flow. In a 3D flow, the velocity varies in all x, y and z directions. The velocity profile formed is parabolic in nature, which allows less amount of maximum mass flow rate in a channel, as compared to that in a slug flow. Due to this less amount of maximum mass flow rate, the flow seems to choke at an exit Mach number lower than that of 1D flow. This explains the reason for subsonic choking observed in our measurements.

4. FORMULAS

$$Re = \frac{\dot{m}D_h}{\mu A_c} \quad (1)$$

$$Kn = \frac{\lambda}{D_h} = \frac{\mu}{pD_h}\sqrt{\frac{\pi RT}{2}} \quad (2)$$

$$Ma = \frac{p}{G\sqrt{\gamma}(RT)^{3/2}} \quad (3)$$

$$f_{exp} = \frac{D_h}{L}(p_i^2 - p_o^2)\left\{\frac{1}{G^2RT} - \frac{1}{p_i p_o}\right\} \quad (4)$$

$$f_{theory} = \frac{D_h}{L}\left(\frac{1-\gamma Ma_i^2}{\gamma Ma_i^2} - \frac{1-\gamma Ma_o^2}{\gamma Ma_o^2} + ln\frac{Ma_i^2}{Ma_o^2}\right) \quad (5)$$

where, Re is the Reynolds number, μ is the dynamic viscosity, Kn is the Knudsen number and λ is mean free path of the nitrogen gas. D_h is the hydraulic diameter of microchannel given by $4A_c/p$. Here, A_c is area of cross-section and p is perimeter. G is mass velocity given by $\dot{m}/A_c$, where $\dot{m}$ is the mass flow rate measured. R is the gas constant, T is the gas temperature, is the ratio of specific heats for the gas. Ma_i and Ma_o are the inlet and outlet Mach numbers, respectively. L denotes the microchannel length and f is the friction factor. p_i, p_o and Δp are the measured inlet pressure, outlet pressure and pressure drop, respectively.

6. CONCLUSIONS

It is concluded that the highly compressible flow in a three-dimensional microchannel experiences choking behavior in early slip regime. The flow behaves like isothermal, since the temperature measurements at the inlet and outlet of microchannel are of same magnitude. The Poiseuille number decreases monotonically with outlet Mach number. The theoretical limiting choking Mach number of isothermal flow is $1/\sqrt{\gamma} = 0.845$. The flow has a tendency to choke before this theoretical limit. Therefore, the flow is claimed to be choked subsonically. The reason for the subsonic choking is attributed to three-dimensional nature of the flow, which allows lower maximum mass flow rate through a channel, compared to one-dimensional flow.

This experimental study could be applied in reducing frictional resistance of high speed rarefied gas flow scenarios, where flow has a tendency to choke. Few examples could be transport of natural gas in pipelines, space applications and micro-electro mechanical systems. The future scope of this work could be making local temperature and pressure measurements in a three-dimensional microchannel, especially at the locations where theoretical isothermal limiting Mach number is claimed.

NOMENCLATURE

Ma	local Mach number
γ	ratio of specific heats for the gas
f	friction factor
Re	Reynolds number
fRe	Poiseuille number
L	length of microchannel (m)
H	height of microchannel (m)
W	width of microchannel (m)
A_c	area of cross-section of microchannel (m^2)
P	perimeter of microchannel (m)
D_h	hydraulic diameter of microchannel (m)
p	absolute pressure measured (Pa)
Δp	pressure drop (Pa)
ρ	density (kg/m^3)
$\dot{m}$	mass flow rate measured (kg/s)
G	mass velocity ($kg/m^2/s$)
R	specific gas constant (J/kg/K)

T absolute gas temperature (K)
μ dynamic viscosity (kg/m/s)
Kn Knudsen number
λ mean free path of the gas (m)
δ uncertainty in the quantity
α aspect ratio of microchannel

Subscripts
i inlet of microchannel
o outlet of microchannel
exp experimental data
theory theoretical

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