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FLOW CHARACTERISTICS OF CHOKED GAS FLOW THROUGH ADIABATIC MICROTUBES

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KEY WORDS

Friction Factor, Mach number, Under-expanded flow.

ABSTRACT

A choked flow has been extensively investigated over the years under the condition that the inlet pressure is preserved at a specific (atmospheric) pressure and the back pressure is decompressed. On the other hand, in the case of atmospheric back pressure and the further increase in inlet pressure, the gas velocity become limited whereas the mass flow rate (Reynolds number) keeps on increasing. In this situation the outlet pressure of the channel is higher than the back pressure and the flow becomes under-expanded. However in the case of microchannel gas flow, details of choked (under-expanded) gas flow at the microchannel outlet are still unrevealed because of measurement limitations.

Fortunately for an adiabatic microchannel gas flow, a gas static temperature estimation at the outlet of a micro-channel can be done using a quadratic equation proposed by Kawashima and Asako [1]. A new data reduction methodology for the average friction factor calculation between the inlet and the outlet considering the effect of a decrease in gas temperature has been developed by Hong et al. [2]. Rehman et al. [3] experimentally and numerically investigated the average friction factor along adiabatic microchannels with compressible gas flows including choking flow regime. They reported that both the assumption of perfect expansion and consequently wrong estimation of average temperature between inlet and outlet of a microchannel can be responsible for an apparent increase in experimental average friction factor in choked flow regime. Kawashima et al. [4] investigated numerically the Mach number and pressure at outlet plane of a straight microtube for both laminar and turbulent flow cases. They found that the Mach number at the outlet plane of the choked flow depends on the tube diameter and ranges from 1.16 to 1.25. Kang et al. [5] experimentally and numerically investigated average friction factor under the situation of choked (under-expanded) gas flow in adiabatic micotubes. In order to maintain adiabatic condition from microtube exterior to the surrounding environment, their microtube exterior was covered with foamed polystyrene. However, the heat loss from the microtube was not evaluated qualitatively even though they obtained friction factor and gas temperature under the assumption of adiabatic condition.

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In this article, we describe a combined analysis for numerically and experimentally obtained average friction factors of choked gas flow through adiabatic microtubes. Experiments are performed for microtubes with 249 and 528.9 mm in diameter, by varying the aspect ratio (i.e. length/diameter) from 100 to 200. Fig. 1 shows a schematic diagram of an experimental setup. The gas used in this experiments is a nitrogen. The microtubes were installed in a vacuum chamber to avoid heat transfers by natural convection from outside of the test section. The chamber was evacuated by a vacuum pump in advance to maintain the inside pressure low. The outer wall temperature was measured by two thermocouples (bare wire type-K, ϕ=50μm) attached to the micro-tube outer wall at two locations along the length with a high conductivity epoxy. An infrared thermo-camera (FLIR Systems, FLIR C3) will be also used to measure the outer wall temperature. In the present study, the measure of adiabatic condition is qualitatively evaluated by measuring wall temperatures since the wall temperature in adiabatic microtube have similar trend with that of the gas temperature strongly depending on gas velocity.

In order to make a comparison with the experimental results, numerical computations based on the Arbitrary-Lagrangian-Eulerian (ALE) method were also conducted for a fused silica tube of D = 249 mm whose boundary conditions are identical to the experimental conditions. In order to capture the under-expansion characteristics of the flow during choking, the computational domain is extended in the downstream region beyond the microtube outlet as shown in Fig. 2. A detailed description of the numerical computation is well documented in the previous work [4]

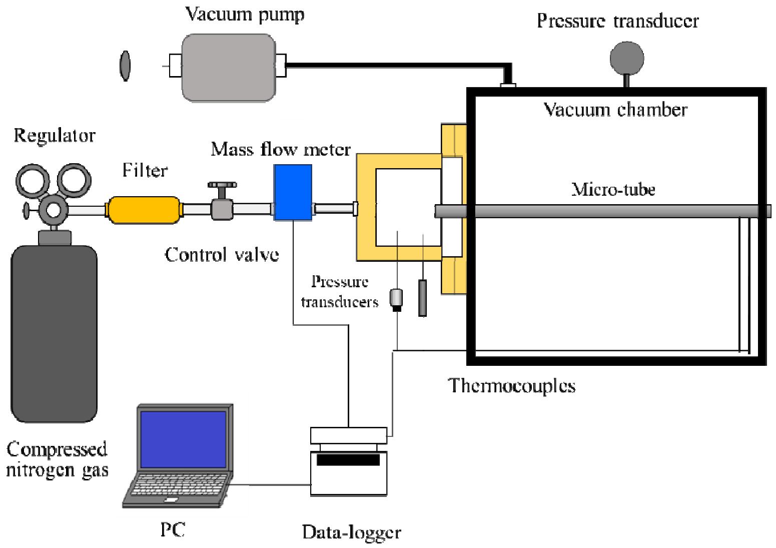


Figure 1: Schematic diagram of experimental setup

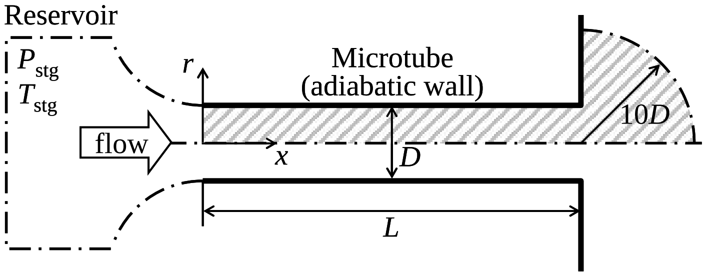


Figure 2: Schematic diagram of a calculation domain

The preliminary experiments were conducted before the test section is installed in a vacuum chamber. The average Fanning friction factors between the inlet and outlet, $f_{\mathrm{f,\ ave}}$ for all tubes were obtained by the following equations [2].

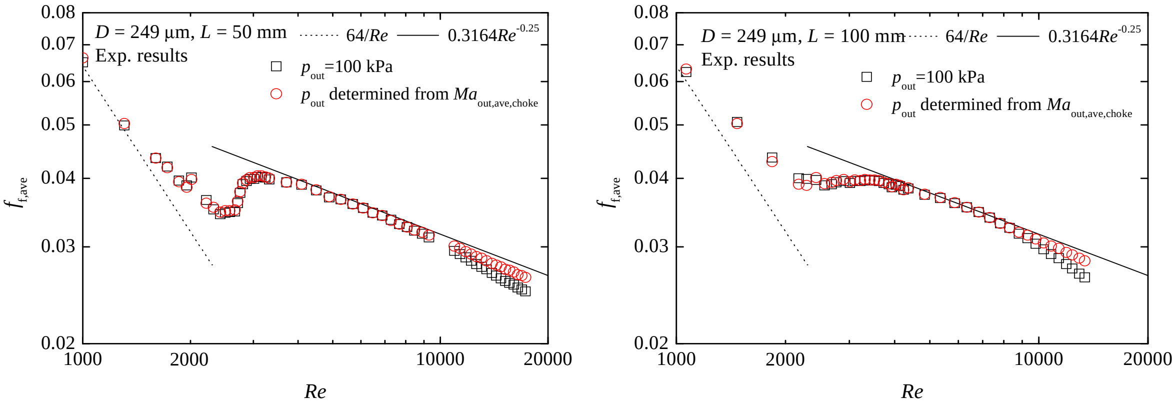
$$f_{\mathrm{f,ave}}=\frac{1}{x_{\mathrm{out}}-x_{\mathrm{in}}}\int_{x_{\mathrm{in}}}^{x_{\mathrm{out}}} f_{\mathrm{f}}\,\mathrm{d}x=\frac{D}{x_{\mathrm{out}}-x_{\mathrm{in}}}\left\{\int_{x_{\mathrm{in}}}^{x_{\mathrm{out}}}\left(\frac{2}{p}\mathrm{d}p\right)-\int_{x_{\mathrm{in}}}^{x_{\mathrm{out}}}\left(\frac{2p}{\rho^2u^2RT}\mathrm{d}p\right)-\int_{x_{\mathrm{in}}}^{x_{\mathrm{out}}}\left(\frac{2}{T}\mathrm{d}T\right)\right\}$$

$$(1)$$

$$=\frac{D}{x_{\mathrm{out}}-x_{\mathrm{in}}}\left[\begin{array}{l}-2\ln\dfrac{p_{\mathrm{in}}}{p_{\mathrm{out}}}+2\ln\dfrac{T_{\mathrm{in}}}{T_{\mathrm{out}}}-\dfrac{1}{\left(\rho_{\mathrm{in}}^2u_{\mathrm{in}}^2R\times\left(T_{\mathrm{in}}+\dfrac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}}\right)\right)}\\ \times\left\{\dfrac{p_{\mathrm{out}}^2-p_{\mathrm{in}}^2}{2}+\dfrac{B^2}{2}\ln\dfrac{p_{\mathrm{out}}+\sqrt{p_{\mathrm{out}}^2+B^2}}{p_{\mathrm{in}}+\sqrt{p_{\mathrm{in}}^2+B^2}}+\dfrac{1}{2}\left(p_{\mathrm{out}}\sqrt{p_{\mathrm{out}}^2+B^2}-p_{\mathrm{in}}\sqrt{p_{\mathrm{out}}^2+B^2}\right)\right\}\end{array}\right]$$

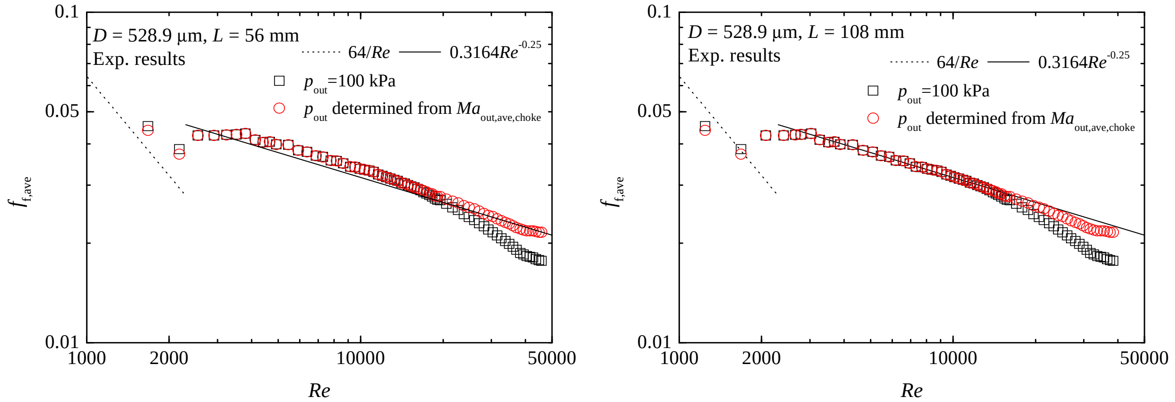
where,

$$B^2=4\times\alpha\frac{\rho_{\mathrm{in}}^2u_{\mathrm{in}}^2R^2}{2c_{\mathrm{p}}}\times\left(T_{\mathrm{in}}+\frac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}}\right) \quad (2)$$



(a) L = 25 mm (b) L = 50 mm

Figure 3: Average friction factor vs Re for D = 249 μm



(a) L = 56 mm (b) L = 108 mm

Figure 4: Average friction factor vs Re for D = 528.9 μm

The values of $f_{\mathrm{f,\,ave}}$ obtained from Eq. (3) are plotted in Figs. 3 and 4 as a function of Reynolds number. The solid line and dotted line in the figures represent the values obtained by the theoretical formula (f = 64/Re) and $f = 0.3164/Re^{0.25}$ (*Blasius* correlation) for the incompressible flow, respectively. As can be seen in the figures the flow transits from laminar flow to turbulent flow in the range of $2000 < Re < 4000$ the same as conventional sized tubes. In the laminar flow regime on Figs 3 (a) and (b), the values of $f_{\mathrm{f,\,ave}}$ deviate more and more from that of an incompressible flow with an increasing in Reynolds number because of the compressibility effect. In the case of the turbulent flow regime before flow choking (unchoked turbulent flow regime) on Figs. 3 and 4, the values of $f_{\mathrm{f,\,ave}}$ nearly coincide with *Blasius* equation. However, in the case of the turbulent flow regime after flow choking (choked turbulent flow regime), the values of $f_{\mathrm{f,\,ave}}$ obtained under the assumption of $p_{\mathrm{out}} = p_{\mathrm{atm}}$ deviate in the lower direction from *Blasius* equation with an increase in Reynolds number since the assumption of $p_{\mathrm{out}} = p_{\mathrm{atm}}$ is not valid for a choked flow. At the outlet, Mach number with the equation of state and mass flow rate per unit area, $\dot{G}$ (kg/(s m^2)) can be rewritten as

$$Ma_{\mathrm{out}} = \frac{u_{\mathrm{out}}}{\sqrt{\gamma R T_{\mathrm{out}}}} = \frac{\dot{G}}{p_{\mathrm{out}}}\sqrt{\frac{R T_{\mathrm{out}}}{\gamma}} \tag{3}$$

And the following equation can be obtained for an adiabatic channel flow

$$T_{\mathrm{out}} = \frac{2T_{\mathrm{stg}}}{(\gamma - 1)Ma_{\mathrm{out}}^2 + 2} \tag{4}$$

Then, the outlet pressure is

$$p_{\mathrm{out}} = \frac{\dot{G}}{Ma_{\mathrm{out}}}\sqrt{\frac{R T_{\mathrm{out}}}{\gamma}} = \frac{\dot{G}}{Ma_{\mathrm{out}}}\sqrt{\frac{2 R T_{\mathrm{stg}}}{\gamma\left[(\gamma - 1)Ma_{\mathrm{out}}^2 + 2\right]}} \tag{5}$$

If the outlet Mach number is given, the outlet pressure and temperature can be determined from equation (4) and (5). Kawashima et al. [4] reported the average Mach number at the outlet plane of the choked flow depends on the tube diameter and proposed a correlation for the average Mach number at the outlet plane of the choked flow as

$$Ma_{\mathrm{out,ave,choke}} = 1.16 \times 10^5 D^2 - 279D + 1.27 \tag{6}$$

Therefore $f_{\mathrm{f,\,ave}}$ was obtained with substituting p_{out} determined by equation (5) into equation (1) The values of $f_{\mathrm{f,\,ave}}$ were also plotted in Figs. 3 and 4 with red symbols. The values on Figs. 3 (a) and (b) is slightly lower than *Blasius* equation and the values on Figs. 4 (a) and(b) almost coincide with *Blasius* equation. As a result of that, when the flow is choked, the gas velocity (Mach number) and gas temperature at the outlet remain unchanged, and the outlet pressure is higher than the back pressure (atmospheric pressure) with an increase in Reynolds number. However, the outlet temperature obtained under the assumption of $p_{\mathrm{out}} = p_{\mathrm{atm}}$ does not remain unchanged rather steeply decreases. Therefore in the choked turbulent flow regime, the arithmetic average gas temperature between the inlet and outlet decreases and $f_{\mathrm{f,ave}}$ decreases.

As mentioned above, $Ma_{\mathrm{out,ave,choke}}$ considering flow choking is a specific value represented as a function of the tube diameter [4]. The outlet pressure determined by $Ma_{\mathrm{out,ave,choke}}$ is higher than atmospheric pressure and the outlet gas temperature determined by it remains unchanged. Then, $f_{\mathrm{f,ave}}$ is slightly lower than *Blasius* equation or nearly coincide with *Blasius* equation.

However, in actual situation, the outlet gas temperature increases or decreases depending on the Joule-Thomson coefficient when an adiabatic (isenthalpic) microtube works by discharging gas into the atmosphere under an increasing inlet pressure. Therefore, flow characteristics of a choked gas flow through adiabatic microtubes in the vacuum chamber will be reported at the conference.

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