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LAMINAR TO TURBULENT FLOW TRANSITION IN A RECTANGULAR DUCT WITH 1:10 ASPECT RATIO EVALUATED USING DNS AND RANS TRANSITIONAL TURBULENCE MODEL

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Abstract

Numerical simulations are performed for a smooth rectangular duct with aspect ratio (height to width) of 0.1 using Direct Numerical Simulations and a two equations, intermittency based $\gamma - Re_\theta$ RANS transitional turbulence model. Statistically averaged velocity and pressure fields are extracted and friction factor curve is produced. Focus has been given to evaluate the capability of utilized transitional turbulence model in respect of predicting the average transitional flow characteristics. To this end friction factor comparison evaluated from the RANS model is compared against DNS results as well as experimental results of a microchannel having the aspect ratio of 0.11. Due to complexities associated to compressibility modeling, for the initial study gas flow in DNS simulations is assumed to be incompressible whereas, compressibility is modeled in RANS based modeling approach. Comparison between the two models and experiments show that employed RANS transitional model although predicts the critical Reynolds number with sufficient accuracy compared to experiments but overestimates the pressure drop during the late transitional regime. As expected, DNS on the other hand follows the experimental results fairly well in transitional regime but shows an abrupt transition compared to experiments. When compared with experimental results, RANS model underpredicts the critical Reynolds number by 6.16% whereas DNS overpredicts it by 7.17%. Slight discrepancies of critical Reynolds number and the behavior of transitional regime in the case of DNS can also be due to the assumption of fully developed flow at the inlet of computational domain compared to experimental inlet manifold and piping details. Velocity flow fields using DNS results and their comparison to RANS model will be done in order to elaborate on the fluid dynamic behavior in transitional regime.

KEYWORDS: pressure drop, friction factor, transitional modeling, microchannel, gas flow

1. INTRODUCTION

Starting with Osborne Reynolds [1] more than a century ago, turbulent transition within wall bounded flows has been investigated mainly through experimental observations. Whereas in recent years DNS has also been applied to understand the physics of flow transition inside tubes [2]. A major breakthrough in modeling transitional flow using two equations turbulence models (RANS) is due to Menter et al. [3]. Although originally developed for external flows, model constants were modified by Abraham et al. [4] to predict transition in internal flows. Current study serves as a first step to investigate and compare the transitional flow characteristics by comparing results from state of the art DNS with a low order RANS model. Such analysis will help to determine limitations of $\gamma - Re_\theta$ RANS turbulence model to predict fluid dynamics behavior of gas flows in transitional regime. This in turn will also help to asses the applicability of this relatively computationally inexpensive RANS model for the product development of micro heat exchangers that may operate in transitional flow regime.

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2. NUMERICAL SETUP

Direct Numerical Simulations (DNS) are performed by ThermALab of Energy Department of Politecnico di Milano. The adopted software is developed at PoliMi for the investigation of channel flows and later adapted to simulate the flow within finite-Aspect Ratio ducts. The code consists in a Compact Finite Difference, structured-grid solver for unsteady, three-dimensional, incompressible Navier-Stokes equations. Time integration is achieved by means of three-step Rai scheme, which was preferred over other schemes since it offers the best compromise between accuracy and robustness. The initialization of the calculation is achieved by imposing the solution for the laminar flow, where the normalized streamwise velocity is known and results from the analytic solution of Navier-Stokes equations, which is possible in the considered geometry. To observe the transition to turbulent-flow regime, if any, a random perturbation is added during the first time steps to the laminar solution so that, if the Reynolds number is high enough, the transitions to turbulence is triggered. Conversely, for low Reynolds numbers, the flow returns to a stable, laminar condition when the perturbation terminates. The parameters of the perturbation must be tuned by means of a delicate trial-and-error procedure, since it must respond to the compromise between the preservation of numerical stability (which is hampered, if the perturbation intensity is too high) and the capability of inducing the physical transition to turbulent conditions. The presence of lateral walls is simulated by imposing zero velocity on both walls in the spanwise direction, to comply non-penetration and no-slip boundary conditions; the derivative of the pressure in the wall normal direction is set to zero. The described simulations are performed under the assumption of fully-developed flow, and therefore periodic boundary conditions in the streamwise direction are applied to both velocity and pressure.

The periodicity on velocity in the streamwise direction is commonly imposed when fully developed flows are simulated. Conversely, the same solution is seldom applied to the pressure field, since it would be equivalent to a null pressure drop across the duct, which implies a meaninglessly-null friction factor. The physics of the flow is preserved by introducing a body force, like e.g. gravity acceleration, which produces a hydrostatic pressure distribution that compensate the pressure drop due to friction. Clearly, since the pressure gradient is considered unknown, either the mass flow rate or the body force must be known in advance in order to adopt this approach. In this work, the mass flow rate is imposed, whereas the driving body force is not known. To achieve the final computation of the pressure field and, at the same time, the prescribed mass flow rate, a multistep procedure is adopted which considers only one of the two unknowns per time. The aforementioned procedure, adopted to achieve the implementation of streamwise periodic boundary conditions on pressure, can be also used to compute the friction factor. The second step of the method, indeed, requires to correct the velocity field at each time step, to preserve the prescribed, constant mass flow rate. This correction corresponds also to a pressure gradient, which can be used to compute the head loss and, therefore, the friction factor. The same result is achieved if the friction factor is computed by means of a balances of forces along the streamwise direction: the forces due to non-normal stresses along the lateral walls (which depend on the wall-normal derivative of the streamwise velocity) must balance the pressure difference due to friction between the inflow and the outflow. A detailed description of employed DNS model can be found in [5].

Implementation of the RANS transitional turbulence model on the other hand is performed using ANSYS CFX®. Due to relatively easy computational modeling than DNS, all the inlet geometric details encountered in experimental assembly are reproduced as shown in Fig. 1. Further details of numerical model implementation can be found in [6]. Considering one dimensional flow of ideal gas, average Fanning friction factor between inlet '*in*' and outlet '*out*' of a MC with hydraulic diameter D_h and length L can be defined by the following expression for a compressible flow [6]:

$$f_f = \frac{D_h}{L}\left[\frac{p_{in}^2 - p_{out}^2}{RT_{av}\dot{G}^2} - 2\ln\left(\frac{p_{in}}{p_{out}}\right) + 2\ln\left(\frac{T_{in}}{T_{out}}\right)\right] \tag{1}$$

where p and T denote cross sectional average pressure and temperature of gas, T_{av} is the average temperature of the gas between inlet and outlet of MC, and $\dot{G}$ is mass flow per unit area ($\dot{G} = \frac{\dot{m}}{A}$).

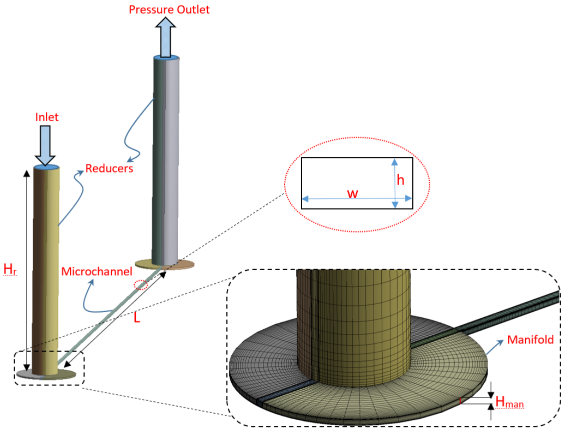

Figure 1: Geometric mesh details for $\gamma - Re_\theta$ RANS model.

3. RESULTS

Laminar to turbulent flow transition is established using the average friction factor curve. Critical Reynolds number (Re_{cr}) is defined as the Reynolds number in correspondence of which the friction factor attains its first minimum and then starts to increase. This point is individuated during post processing of numerical results. Numerical results of friction factor from both DNS and RANS models are compared against experimental results (taken from [7]) of the MC with $\alpha = 0.11$ in Fig. 2. There exists an excellent agreement between the DNS and experimental results in the laminar as well as fully turbulent flow regimes where f_f in the laminar and turbulent flow regimes are compared with Shah & London correlation (S&L) and Blasius correlations respectively:

$$\begin{aligned} S\&L : f_{f_{SL}} &= \frac{96}{Re}\left(1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5\right) \\ Blasius : f_{f_{BL}} &= 0.3164 Re^{-0.25} \end{aligned} \tag{2}$$

Both numerical models can predict the laminar frictional behavior with exceptional accuracy which comes as a no surprise as gas flow inside the duct is two dimensional without any secondary flows. In turbulent regime experimental results are better estimated by DNS compared to $\gamma - Re_\theta$ RANS turbulence model which overestimates the experimental pressure drop with chosen values of model constants. Re_{cr} is estimated to be ∼2222 with RANS model, ∼2545 by DNS whereas it is ∼2368 from experimental results. Transition is slightly delayed with DNS, 7.17% compared to experiments, which can be due to two reasons. First, to reduce the computational domain of DNS model, only a small portion of the duct length is modeled with periodic boundary condition in the streamwise direction and secondly experimental details of duct inlet piping are ignored as well. On the contrary, all these details are catered for in the case of RANS model and yet transition is 6.16% anticipated compared to experiments. Both numerical models show a steep or an abrupt transition to the fully developed turbulent flow with RANS model going the highest above the Blasius. DNS evaluated f_f recovers itself and is within the experimental uncertainty in turbulent regime whereas RANS model overpredicts the fully turbulent f_f throughout the experimental range of Re. It is worth mentioning at this point that model constants recommended by the Abraham et al. [4] have been modified in the current work to calibrate the model such that Re_{cr} is close to experimental results. Details of model calibration as well as effect of secondary flows on the friction factor, evaluated from DNS will be discussed.

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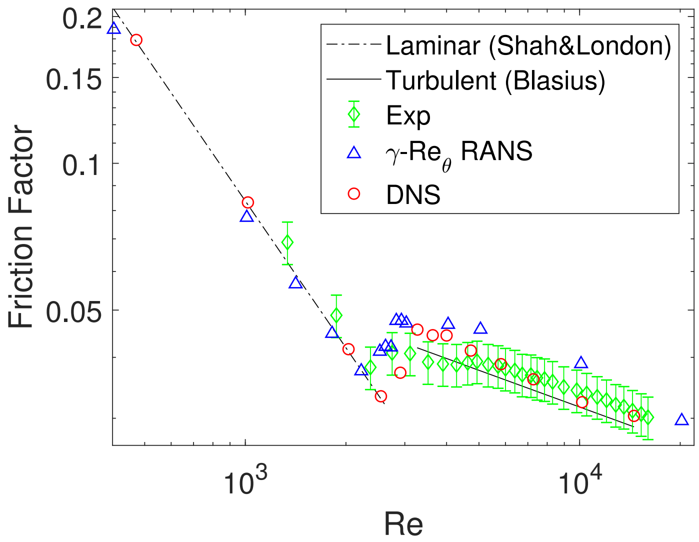

Figure 2: Friction factor comparison between DNS, RANS and experimental results.

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